

PERIODICITY FORMULAS:

Sidereal Orbit	$(365.25636042 + 1.1 \times 10^{-7} \text{ TE}) \text{ days}$
Tropical Year	$(365.24219878 - 6.14 \times 10^{-6} \text{ TE}) \text{ days}$
Eclipse Year	$(346.620031 + 3.2 \times 10^{-5} \text{ TE}) \text{ days}$
Anomalistic Year	$(365.25964134 + 3.04 \times 10^{-6} \text{ TE}) \text{ days}$
Sidereal Lunar Orbit	$(27.3216610 - 2.0 \times 10^{-7} \text{ T}) \text{ days}$
Lunar Mean Daily Sidereal Motion	$(13.1763582975 - 1.0224 \times 10^{-8} \text{ T})^\circ$
Lunar Synodical Period	$(29.5305992 - 2.0 \times 10^{-7} \text{ T}) \text{ days}$
Centennial General Precession Longitude	$(1.396291666... + 0.0006180555... \text{T})^\circ$

Given TE = Julian centuries from day 0.5, 1900 ET

Given T = Tropical centuries from 1900.0 N

DEFINITIONS:

Sidereal Orbit	is a revolution relative to a fixed celestial position.
Sidereal Noon	is the instant of transit of mean equinox relative to a fixed meridian position.
Fundamental Epoch of Sidereal Time	(FE) is the instant 12 hours, 0 days, 1900 years A.D. with hours in mean sidereal time.
Ephemeris Time	is the actual count of solar days from a fixed meridian.
Tropical Year (YT)	is the period from equinox to equinox.
Eclipse Year (YE)	is the period between the earth and lunar orbit planes node crossings.
Temporal Unit (TU)	is 36,525 mean solar days since Jan. 0.5, 1900, UT.
Greenwich Mean Sidereal Time (GT)	= 0.0 hours UT = 12 hours + aFMS
Universal Time (UT)	has replaced Mean Solar Time due to a recognition of the non-uniform rotation rate of the earth.
Lunar Synodic Period (S9)	is the period of time from one full or new moon to another, that is the time between consecutive alignments of the sun, earth and moon on a plane perpendicular to the plane of solar revolution.
Precession (PR)	is the retrograde rotation of the earth's axis relative to fixed celestial reference.
Annual Parallax	is the viewpoint difference due to the change in the earth's position relative to the sun. For the nearest star the angle is about 0.000222° .
Annual Aberration	is the angular shift in apparent position resulting from motion velocity of viewing from orbiting (moving) earth.
Durnal Parallax	is the viewpoint difference due to the rotation of the earth. The amount varies with the latitude of the observer.
Durnal Aberration	is the result of observing from a spinning observing position on the surface of the earth. Velocity of the observer causes apparent shift to a maximum correction of about 0.0008333° at the equator.
Atmospheric Refraction	is the bending of light rays by the earth's atmosphere.

NOTATION

aFMS	Fictitious Mean Solar position
DMS	Day, Mean Sidereal
d, h, m, s,	day, hour, minute, second
ES	Ephemeris Second
ET	Ephemeris Time
FE	Fundamental Epoch
GT	Greenwich Mean Sidereal Time
JC	Julian Century
JD	Julian Day
L°	Longitude of the Mean Sun
R°	Period of Sidereal Rotation
T	Tropical Centuries from 1900.0 N
TE	Temporal Epoch
TU	Temporal Solar Based Unit
UT	Universal Time

TIME FORMULAS

aFMS	$0.776919398148d + 8640184.s628 \text{ TU} + 0.0929 \text{ TU}^2$
DMS	86,400.s
DMS / P°	$0.999999902907 - 5.9 \times 10^{-11} \text{ TE}$
DMS / P°	$(1.000000097093 + 5.9 \times 10^{-11} \text{ TE})^{-1}$
ES	Tropical Year 1900 / 31,556,925.9747
FE	12h 0d 1900 A.D. (hours in mean sidereal time)
FE	geometric mean solar longitude : mean equinox @ 279.6966777...°
GT	12h + aFMS
GT	$0.279057325232d + 8640184.s8138 \text{ TU} + 0.s0929 \text{ TU}^2$
JC	36,525 days ephemeris time
L°	$297.69667777...^\circ + 36.001.2914583^\circ \text{ TE} + .0003025^\circ \text{ TE}^2$
Mean Solar Day : Mean Sidereal Time	$1.002737909265 + 5.89 \times 10^{-11} \text{ TU}$
Mean Solar Day : Mean Sidereal Time	$(0.997269566414 - 5.86 \times 10^{-11} \text{ TU})^{-1}$
TE	One JC from 0.d5, 1900 (JD 2,415,020.)
TU	36,525 mean solar days from 12h, Jan. 0, 1900 UT.
TU	36,525 mean solar days from 12h, Jan. 0, 2000 UT1.

ILLUMINATION GEOMETRY

Apparent solar rise-set position given:

L = latitude

A = Azimuth

d = declination

dC = degrees Celsius

dK = degrees Kelvin = dC + 273

h = solar angular elevation

ho = Horizon angle

ha = apparent height of the celestial body considered

p = atmospheric pressure

Apparent solar rise-set position =

$$\cos A = (\sin d - \sin L \sin h) / (\cos L \cos h)$$

Fit of the direction of a site alignment to the rising or setting of the sun. Given a level horizon:

$$\cos A = -\sin d / \cos L$$

Horizon elevation above mathematical level changes Azimuth by the angle A as follows:

$$A = \arcsin \{ \tan ho / [(\cos d / \sin L)^2 - 1]^{1/2} \}$$

Horizon illumination is affected by refraction. The variables of atmospheric pressure and temperature are included according to the following:

$$ha = ho + (a / b)$$

where:

$$a = p (0.1549 + 0.0196 ho + 0.00002 ho^2)$$

$$b = (273 + dC) (1 + 0.5050 ho + 0.0845 ho^2)$$

Solving refraction correction for ho when ha is known results in:

$$ho^3 + a_1 ho^2 + a_2 ho + a_3 = 0$$

Given the coefficients:

$$a_1 = 5.9763 - ha + 0.00023669 (p / dK)$$

$$a_2 = 11.8343 - 5.9763 ha + 0.2320 (p / dK)$$

$$a_3 = -11.8343 + 1.8864 (p / dK)$$

Then reduced by linear transformation:

$$ha + (a_1 / 3) = y \text{ results in } y^3 + p y - q = 0 \text{ with}$$

$$p = a_2 - (a_1^2 / 3)$$

$$q = (a_1 / 3) [(2 a_1^2 / 9) - a_2] + a_3$$

Thus the actual horizon elevation angle can be calculated as follows:

$$ho = [- (q / 2) + D^{1/2}]^{1/3} + [(q / 2) - D^{1/2}]^{1/3} - (a_1 / 3)$$

$$\text{where } D = (q / 2)^2 / (p / 3)^3$$

MISCELLANEOUS FORMULAS AND DATA

Lunar Standstills can be determined by calculation of the longitude of the ascending node of lunar orbit.

The formula is:

$$259.183^\circ - 0.05295^\circ d. + 0.002078^\circ T^2 + 0.000002^\circ T^3$$

given d and T are days and Julian centuries from JD 2,415,020 (Jan. 1, noon, 1900 Ephemeris Time).

Anomalistic Month, the period of the moon's eccentricity of orbit, from perigee to perigee, is 27.5545465 days. Because the moon's distance is least at perigee, parallax is then greatest.

Parallax is produced by viewing the moon from a moving earth. The direction in which the moon appears is determined by position on the earth's surface. To adjust the angle of view on the surface to what it would be from the center of the earth the following formula is used. The lunar parallax correction is greatest when the moon is on the horizon, and zero when the moon is at zenith.

$$p = 0.95075^\circ + 0.0518055555^\circ \cos I + 0.0078333333^\circ \cos M + 0.0095277777^\circ \cos (I - M) + 0.0028333333^\circ \cos 2I + 0.0008611111^\circ \cos (I + M)$$

where I = mean anomaly, mean position of the moon measured counterclockwise from perigee, and

M = longitude of the Moon.

Obliquity of the Ecliptic is the temporally variated angle of the axis of rotation of the earth relative to the plane of revolution around the sun.

$$OB = 23.4392911111^\circ - 0.0130041666...^\circ T - 0.00000163888...^\circ T^2 + 0.0000005036111...^\circ T^3$$

given: T = Julian centuries (36,525 days) from 2000.0.

Annual Secular Polar Motion is the alteration in the position of the axis of rotation relative to the surface of the geoid.

$0''.0035$ ($= 0.00000972222....^\circ$) along the meridian $65^\circ W$.

Centennial General Precession is the slow and gradual retrograde rotation in the direction of the axis of rotation in fixed space.

longitude $= (1.396291666... + 0.0006180555... T)^\circ$

right ascension, $m = (1.280397222... + 0.000777... T)^\circ$

declination, $n = (0.5569666... + 0.000236111... T)^\circ$

Anomalistic Year. The disturbing effects of the planets on the line of the apsides of the earth results in an eastward advance of about 0.032365925° per orbit, thus producing the longer anomalistic year of $(365.25964134 + 3.04 \times 10^{-6} TE)$ days

Ephemeris Time

The Ephemeris Second, the unit of measure of Ephemeris Time, is defined in terms of the tropical year 1900, the fundamental ephemeris epoch. The tropical year 1900 = 31,556,925.9747 seconds.

Velocity of Light (in a vacuum)

$c = 299,792,458$ meters per second = 186,282.4 miles per second.

Astronomical Unit = AU = 149,597,870 km

Solar Parallax = $0.00244281888...^\circ$

Solar Radius = 696,000 km

Gaussian Gravitational Constant = $k = 0.01720209895$

$$\text{Constant of Gravitation} = G = 6.672 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$\text{Geocentric Gravitational Constant} = 3.9860310 \times 10^{14} \text{ m}^3 \text{ s}^2$$

$$\text{Heliocentric Gravitational Constant} = 1.32712438 \times 10^{20} \text{ m}^3 \text{ s}^2$$

$$\text{Mass of the Sun} = 1.9891 \times 10^{30} \text{ kg}$$

$$\text{Mass of the Earth} = 5.974 \times 10^{24} \text{ kg}$$

$$\text{Mass of the Moon} = 7.348031948 \times 10^{22} \text{ kg}$$

$$\text{Constant of Nutation} = 0.00255625^\circ$$

$$\text{Constant of Aberration} = 0.0056932$$

$$\text{Mean Longitude of the Sun} = 279.696677778^\circ + 36,000.768925^\circ \text{ TE} + 0.0003025^\circ \text{ TE}^2$$

Saros Cycle is an eclipse cycle of 242 nodal months, 223 synodic periods, 239 anomalistic months and 17 eclipse years.

$$242 \times 27.21222 = 6585.357425$$

$$223 \times 29.53059 = 6585.321321$$

$$239 \times 27.55455 = 6585.536614$$

$$19 \times 346.62006 = 6585.781197$$

Metonic Cycle is a cycle that produces the same phase of the moon on the same date of the tropical year every 19 years.

$$235 \times 29.53059 = 6939.688388$$

$$255 \times 27.21222 = 6939.116295$$

$$19 \times 365.24219 = 6939.601660$$

SOME PLANETARY DATA

Planet	Equatorial Radius (km)	Orbital Period in days	Semi-major axis of orbit in AU	Eccentricity
Mercury	2439	87.97046	0.387099	0.205629
Venus	6051	224.69815	0.723326	0.006772
Earth	6378.14	365.25636053	1.000018	0.016773
Mars	3393.4	686.9257	1.523638	0.093298
Jupiter	71,398	4332.23025	5.20248	0.048058
Saturn	60,000	10,800.4425	9.56329	0.050916
Uranus	25,400	30,953.4765	19.2937	0.047285
Neptune	24,300	60,839.6925	30.2743	0.006851
Pluto	1500	91,305.195	39.6823	0.252786

COSMOGRAPHIC VALUES INDEX

A SET OF COSMOGRAPHIC VALUES WITH CODE

CONSTANT	NUMERIC VALUE	CODE
Earth, Radius, Equatorial	3963.19245606 mi. 6,378,140 m	ER
Earth, Circumference, Equatorial	24,901.4726094 mi. 40,075,035.5351 m	CE
Earth, Circumference, Mean	24,873.492365 mi. 40,030,005.6967 m	CRM
Earth, Arc Degree, Mean	69.0933962964 mi. 111,1950.42769 m	AM
Earth, Radius, Mean	3958.73926185 mi. 6,370,973.27862 m	RA
Earth, Radius, Polar	3949.90462476 mi. 6,356,755.28816 m	RP
Flattening, axis ratio	0.996647186822	FL
Flattening inverse	298.257	FI
Flattening of the Earth	0.0033528131779	FE
Inclination, Lunar Orbit, Mean	5.1453964°	IL
Wobble, Lunar Orbit Inclination	± 0.0025°	WL
Lunar Major (OB + IL)	28.584687511°	OL
Lunar Minor (OB - IL)	18.293894711°	OM
Moon, Distance	384,399,070 m	MD
Lunar Orbit Eccentricity	0.054900489	LE
Moon, Radius, Mean	1,738,000 m	MR
Obliquity of the Ecliptic	23.439291111°	OB
Solar distance	149,597,870,000 m	AU
Geographical Mile	1,855.32571922 m	MG
MODULES	in degrees	CODE
Lunar Motion per Mean Rotation	13.1403824445°	R27
Mean Daily Lunar Motion	13.1763582244°	C27
Solar Orbit per Day	0.985609119791°	SO
Solar Orbit per Sidereal Rotation	0.982918083604°	SI
Solar Orbit per Nodal Month	26.8206129544°	S22
Solar Orbit per Sidereal Month	26.9284788014°	S27
Solar Orbit per Synodic Month	29.1056177173°	S29

ASTRONOMIC CONSTANTS INDEX

A SET OF ASTRONOMIC CONSTANTS WITH CODE

CONSTANT	NUMERIC VALUE	CODE
Earth, Orbit, Days	365.25636053	DO
Earth, Orbits per Day	0.00273780311053	OD
Earth, Orbits per Lunar Orbit	0.0748013300039	OL
Earth, Orbits per Nodal Month	0.0745017026513	ON
Earth, Rotations, Lunar Orbit	27.39646289	RL
Earth, Rotations, Nodal Month	27.2867224663	RN
Earth, Rotations, Orbit	366.25636053	RO
Earth, Rotations, Synodic Month	29.6114378225	R9
Earth, Rotations, Year	366.242154402	RY
Eclipse Year, Days	346.620063	YE
Eclipse Year, Rotations	347.569040486	RE
Lunar Nodal Month, Days	27.2122207637	S2
Lunar Nodal Months per Day	0.0367481951835	ND
Lunar Nodal Months per Orbit	13.4225120288	NO
Lunar Nodal Months per Rotation	0.0366478605569	NR
Lunar Orbit, Days	27.32166156	S7
Lunar Orbits per Day	0.0366009950677	LD
Lunar Orbits per Orbit	13.3687462502	LO
Lunar Orbits per Rotation	0.0365010623457	LR
Lunar Sidereal Month, Days	27.32166156	S7
Lunar Synodic Month, Days	29.5305888844	S9
Nutation Cycle, Orbits	18.6140238945	NUO
Nutation Cycle, Years	18.6133019052	YN
Precession Cycle, Years	25781.5756912	PR
Rotation, Seconds	86636.5461887	SRO
Sidereal Day, Seconds	86164.099	SDS
Solar Day, Seconds	86400	SD
Year, Eclipse, Days	346.620063	YE
Year, Rotations	366.242154403	YR
Year, Tropical, Days	365.24219264	YT
Years per Day	0.002737909311	YD
Years per Lunar Orbit	0.0748042315774	YL
Years per Nutation	18.6133019052	YN
Years per Orbit	0.0999961212611	YO

Solar System

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Solar Radius	696,000		

Astronomical Unit = **AU** = 149,597,870 km

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Earth and Moon

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Periodicity

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$$\begin{aligned} 242 \times 27.21222 &= 6585.357425 \\ 223 \times 29.53059 &= 6585.321321 \\ 239 \times 27.55455 &= 6585.536614 \\ 19 \times 346.62006 &= 6585.781197 \end{aligned}$$

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Periodicity Formulas

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GRAVITY

Mass of the Sun = 1.9891×10^{30} kg
Mass of the Earth = 5.974×10^{24} kg
Mass of the Moon = $7.348031948 \times 10^{22}$ kg
